[Q.1] Find the maximum and minimum values of the polynomial $8x^5 - 15x^4 + 10x^2$.

Soln.

Let $f(x) = 8x^5 - 15x^4 + 10x^2$ — ①

taking first order derivative of $f(x)$

$$f'(x) = 40x^4 - 60x^3 + 20x$$

or $f'(x) = 20x(2x^3 - 3x^2 + 1)$

$$= 20x(x-1)^2(2x+1)$$

Now taking $f'(x) = 0$, we obtain the soln. of $x(x-1)^2(2x+1) = 0$ as $x = 0, 1, -\frac{1}{2}$

Now taking second order derivative of $f(x)$, we obtain

$$f''(x) = 160x^3 - 180x^2 + 20$$

or $f''(x) = 20(8x^3 - 9x^2 + 1)$ — ②

Now $f''(-\frac{1}{2}) = -45 < 0$

therefore $f(-\frac{1}{2}) = 8(-\frac{1}{2})^5 - 15(-\frac{1}{2})^4 + 10(-\frac{1}{2})^2$

$$= \frac{-4}{24} - \frac{15}{24} + \frac{40}{24}$$

$$= \frac{21}{24}$$

or $f(-\frac{1}{2}) = \frac{21}{16}$ is a maximum value.

Again $f''(0) = 20 > 0$, therefore

$f(0) = 0$ is a minimum value

Next, $f''(1) = 20(8-9+1) = 0$

$\because f''(1) = 0$, we examine $f'''(1)$

$f'''(x) = 480x^2 - 360x$

Thus $f'''(1) = 480 - 360 = 120 \neq 0$

Therefore $f(1)$ is neither a maximum nor a minimum value

H.W.

Q.1 Find the maximum and minimum values of

$x - \sin 2x + \frac{1}{3}\sin 3x$ in $-\pi \leq x \leq \pi$.

Q.2 Find the maximum value of $(x-1)(x-2)(x-3)$.

Q.3 Show that x^x is minimum for $x = e^{-1}$.

Q.4 Find the maximum and minimum value of the function $f(x) = \sin x + \cos 2x$.

Q.5 Find the values of x for which $\sin x - x\cos x$ is a maximum or a minimum.